Symmetry and entropy of black hole horizons

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Abstract

We argue, using methods taken from the theory of noiseless subsystems in quantum information theory, that the quantum states associated with a Schwarzchild black hole live in the restricted subspace of the Hilbert space of horizon boundary states in which all punctures are equal. Consequently, one value of the Immirzi parameter matches both the Hawking value for the entropy and the quasi normal mode spectrum of the Schwarzchild black hole. The method of noiseless subsystems allows us to understand, in this example and more generally, how symmetries, which take physical states to physical states, can emerge from a diffeomorphism invariant formulation of quantum gravity.

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1 Introduction

This paper is concerned with two distinct problems in background independent formulations of quantum gravity. First, how one can recover the semiclassical Bekenstein-Hawking results[1, 2] in the context of results about the entropy of horizons and surfaces in loop quantum gravity[3]-[6]. Second, how the description of spacetime in terms of classical general relativity emerges from the quantum theory. Up till now, these problems have been treated in isolation. Here we propose a perspective that relates them, and also addresses a technical issue concerning the black hole entropy.

This technical issue concerns the value of a certain free parameter, γ, in the theory called the Immirzi parameter [7]. This parameter, not present in the classical theory, arises from an ambiguity in the quantization procedure because there is a choice in the connection variable that is used in the contruction of the quantum obserable algebra. As it does not appear in the classical theory, its value should be fixed by some physical requirement. In previous work [3]-[5], it was shown that γ is fixed by the requirement that the quantum gravity computation reproduce the Hawking value of the entropy.

Since the story is very simple and physical, we review it for non-experts. The Immirzi parameter comes into the formula for the area of a surface. When a surface is punctured by a set of spin network edges with spin labels $\{j_\alpha\}$, then the area of the surface is given by

$$A[\{j_\alpha\}] = 8\pi\gamma \sum_\alpha \sqrt{j_\alpha(j_\alpha+1)}, \tag{1}$$

which we take here to be measured in units of the Planck area l_{Pl}^2.

In loop quantum gravity, there is an exact description of the quantum geometry of black hole horizons[5], as well as a more general class of boundaries[3]. The entropy of the horizon is defined in terms of the Hilbert space of the boundary theory. For reasons that we spell out shortly, it is taken to be the logarithm of the dimension of the Hilbert space of boundary states.

The Immirzi parameter does not come into the entropy. Hence it appears in the ratio of the entropy $S[A]$ of a black hole to its area:

$$\mathcal{R} = \frac{S[A]}{A[\{j_\alpha\}]} = \frac{c}{\gamma}, \tag{2}$$

where c is a constant to be determined by calculating the entropy. At the same time, we believe from Hawking's semiclassical calculation of black hole entropy that[2]

$$\mathcal{R} = \frac{1}{4}. \tag{3}$$

For the two equations to agree,

$$\gamma = 4c. \tag{4}$$

Recently, one of us, following a clue uncovered by Hod[8], described a semi-classical argument which fixes γ by an argument that appears to be independent of considerations of entropy or radiation but rather makes use of the quasinormal mode spectrum[6]. As it is relevent, we repeat here the basic argument.

The quasinormal mode spectrum of the Schwarzchild black hole turns out to have an asymptotic form

$$M\omega = \frac{\ln(3)}{8\pi} + \frac{i}{4}\left(n + \frac{1}{2}\right), \tag{5}$$

for arbitrarily large integer n. As n goes to infinity, the decay times of the modes goes to zero in units of the light crossing time of the horizon. This means that the excitations involve more and more local regions of the horizon. At the same time, the real part of the frequency goes to the asymptotic value

$$\omega_{qnm} = \frac{\ln(3)}{8\pi M}. \tag{6}$$

This frequency is then associated with an arbitrarily short lived, and hence local, excitation of the horizon. By the correspondence principle, this must correspond to an energy

$$\Delta M_{qnm} = \hbar\omega_{qnm} = \frac{\ln(3)\hbar}{8\pi M}. \tag{7}$$

In the classical theory, the region of the horizon excited may be arbitrarily small, but in the quantum theory there is a smallest region that can be excited, which is a minimal puncture with minimal area

$$\Delta A = 8\pi\gamma\sqrt{j_{min}(j_{min}+1)}. \tag{8}$$

However, for a Schwarzchild black hole,

$$A = 16\pi M^2, \tag{9}$$

from which it follows that, for a quasinormal mode,

$$\Delta A_{qnm} = 32\pi M \Delta M = 4\ln(3) l_{Pl}^2. \tag{10}$$

The result is a prediction for the Immirzi parameter, which is

$$\gamma^{qnm} = \frac{\ln(3)}{2\pi\sqrt{j_{min}(j_{min}+1)}}. \tag{11}$$

Does this match the value needed to get $\mathcal{R} = 1/4$? It turns out to depend on what we take for the Hilbert space of the black hole horizon. By arguments given

in [5], we know that the horizon Hilbert space, for fixed area A in Planck units, is related to the Hilbert space of the invariant states of $U(1)$ Chern-Simons theory on an S^2 with punctures $\{j_\alpha\}$, denoted by $\mathcal{V}_{\{j_\alpha\}}$[1]. The level k is related to the mass of the black hole, and is assumed here to be large. Given a set of punctures $\{j\}$,

$$\mathcal{V}_{\{j_\alpha\}} = \mathrm{Inv}\left(\prod_\alpha \mathcal{H}_{j_\alpha}\right) \tag{12}$$

where $\mathcal{H}_j$ is the $2j+1$ dimensional spin space for spin j and Inv means the Hilbert states contains only states with total spin zero.

We know that the Schwarzchild black hole has definite area $A[\{j\}]$ and definite energy:

$$M(\{j\}) = \sqrt{\frac{A(\{j\})}{16\pi}}. \tag{13}$$

As the area is quantized, we see that the energy is quantized as well.

We then expect that at least some of the Hilbert spaces $\mathcal{V}_{\{j_\alpha\}}$ contain states which correspond to Schwarzchild black holes. The state is expected to be thermal, because it is entangled with the states of radiation that has left the black hole. We are, howeve,r ignorant of the exact state so, instead, the black hole entropy is usually taken to be

$$S^{\text{Schwarzschild}} = \ln \dim \mathcal{H}_A^{\text{Schwarzschild}}, \tag{14}$$

where $\mathcal{H}_A^{\text{Schwarzschild}}$ is a Hilbert space on which the density matrix $\rho_A^{\text{Schwarzschild}}$ describing the ensemble of Schwarzschild black holes may be expected to be non-degenerate.

One proposal is to take

$$\mathcal{H}_A^{\text{Schwarzschild}} = \mathcal{H}_A^{\text{All}} = \sum_{A(\{j\})=A} \mathcal{V}_{\{j\}} \tag{15}$$

where we use the superscript All to remind us that in this case we sum over all sets of punctures that give an area between A and $A + \Delta A$, for some small ΔA.

However, this proposal can be criticized on the basis that the only information about the Schwarzchild black hole that is used is its area. We may expect that the actual state of the Schwarchild black hole is determined by additional physical considerations and hence is non-degenerate only in a subspace

$$\mathcal{H}_A^{\text{Schwarzschild}} \subsetneq \mathcal{H}_A^{\text{All}} \tag{16}$$

that is picked out by additional physical input.

[1]There is an equivalent description in terms of $SU(2)$ Chern-Simons theory[3].

In this article, we will argue that the physical Hilbert space associated to the horizon of a Schwarzschild black hole is dominated by the $\mathcal{V}_{\{j\}}$ for which all punctures have equal minimal spin j_{min}. The difference is important, because in the first case we have

$$\mathcal{R}^{\text{All}} = \frac{\ln \dim \mathcal{H}_A^{\text{All}}}{A(\{j\})}, \tag{17}$$

from which recent calculations[10] have deduced the value[2]

$$\gamma^{\text{All}} = 0.23753295796592\ldots. \tag{18}$$

On the other hand, in the second case

$$\mathcal{R}^{\text{min}} = \frac{\ln \dim \mathcal{V}_{\{j_{min},\cdots,j_{min}\}}}{A(\{j\})}. \tag{19}$$

This is easy to compute [3, 6] and leads to a value

$$\gamma^{\text{min}} = \frac{\ln(3)}{2\pi\sqrt{j_{\text{min}}(j_{\text{min}}+1)}}. \tag{20}$$

We note that

$$\gamma^{\text{qnm}} = \gamma^{\text{min}} \neq \gamma^{\text{All}}. \tag{21}$$

Hence, we have the following situation:

If indeed the right choice for the Hilbert space of a Schwarzchild black hole consists only of states with equal and minimal punctures, then there is a remarkable agreement between the two computations. This supports the case that there is something physically correct about the description of black holes in loop quantum gravity.

On the other hand, if $\mathcal{H}_A^{\text{All}}$ is the right Hilbert space for the horizon of a Schwarzchild black hole, loop quantum gravity is in deep trouble, because there is no choice of γ that will agree with both the Hawking entropy and the quasi-normal mode spectrum.

It is clear that to resolve this problem we need additional physical input. The only property of the Schwarzchild black hole used in previous work to constrain the corresponding quantum state is its area. But a more complete treatement should take into account other characteristics of the Schwarzchild black hole such as its symmetry and stability. To do so, we need to understand how those classical properties can emerge from a quantum state, described so far in the background independent langauge of loop quantum gravity.

Recently, one of us proposed a new perspective on the general problem of the emergence of particles and other semiclassical states from background independent approaches to quantum gravity[13]. This makes use of the concept of *noiseless*

[2]But see [17] for a criticism of that calculation.

subsystems, developed in the context of quantum information theory to describe how particle-like states may emerge there[12]. We shall see in the following that the black hole problem provides a nice example of the general strategy proposed in [13], while it resolves the present problem[3].

2 Symmetry and noiseless subsystems

We begin by asking two questions:

1. How do we find the subspace $\mathcal{H}_A^{\text{Schwarzschild}}$ corresponding to a Schwarzchild black hole, as opposed to a general surface of a given area that satisfies the appropriate boundary conditions?
2. How do we recognize in that subspace the excitations that, in the classical limit, correspond to the quasinormal modes?

We should here make an important comment, which clarifies the sense in which these questions are asked. We note that the boundary conditions we require for our analysis are more general and apply to all horizons, not just Schwarzchild black holes. Thus, our framework differs from the isolated horizon framework[5] in one crucial aspect. We posit that the states of all quantum black holes live in a single Hilbert space, so that angular momentum and other multiple moments are to be measured by quantum operators. In the isolated horizon picture the angular momentum and multiple moments are treated classically, and the Hilbert space is defined for each value of them. Only in the our framework does it make sense to ask how the states corresponding to a non-rotating black hole emerge from a more general Hilbert space of horizon states.

Our two questions are then analogous to problems concerned with the emergence and stability of persistent quantum states in condensed matter physics and quantum information theory. For the present purposes, we will use the following idea from the theory of noiseless subsystems in quantum information theory.

We have a complete quantum experiment, which we want to divide into the system $\mathcal{S}$ and environment $\mathcal{E}$. We want to understand what quantum properties of the system may survive stably in spite of continual and uncontrollable interactions with the environment.

The joint Hilbert space decomposes into the product of system and environment,

$$\mathcal{H}^{total} = \mathcal{H}^{\mathcal{S}} \otimes \mathcal{H}^{\mathcal{E}}, \tag{22}$$

while the Hamiltonian decomposes into the sum

$$H = H^{\mathcal{S}} + H^{\mathcal{E}} + H^{\text{int}} \tag{23}$$

[3]While this paper was in draft, a paper was posted that reaches a similar conclusion by a different argument [11]. More recently, additional arguments for it have been given by [18].

where $H^{\mathcal{S}}$ acts only on the system, $H^{\mathcal{E}}$ acts only on the environment and all the interactions between them are contained in H^{int}.

The reduced dynamics of $\mathcal{S}$ is given by a completely positive operator $\phi : \mathcal{H}^{\mathcal{S}} \to \mathcal{H}^{\mathcal{S}'}$,

$$\rho_f^{\mathcal{S}} = \phi\left[\rho_i^{\mathcal{S}}\right] = \mathrm{tr}_{\mathcal{E}}\left[U\left(\rho_i^{\mathcal{S}} \otimes \rho_i^{\mathcal{E}}\right)U^{\dagger}\right], \tag{24}$$

where the joint state of $\mathcal{S}$ and $\mathcal{E}$ evolves unitarily and then the environment is traced out. While a generic "noise" ϕ will affect the entire state space $\mathcal{H}^{\mathcal{S}}$, there may be a *noiseless subsystem* of $\mathcal{S}$, and possibly even a subspace of $\mathcal{H}^{\mathcal{S}}$ that is left unchanged by ϕ, i.e., evolves unitarily. What follows is a necessary and sufficient condition for the existence of a noiseless subsystem.

Equation (24) can be rewritten as

$$\phi\left[\rho^{\mathcal{S}}\right] = \sum_k A_k \rho^{\mathcal{S}} A_k^{\dagger} \tag{25}$$

where

$$\langle\psi_i|A_k|\psi_f\rangle = \langle\psi_i| \otimes \langle e_k|U|\psi_f\rangle \otimes |e_k\rangle, \tag{26}$$

for any $|\psi_i\rangle, |\psi_f\rangle \in \mathcal{H}^{\mathcal{S}}$ and $\{|e_k\rangle\}$ an orthonormal basis on $\mathcal{H}^{\mathcal{E}}$. One can check that $\sum_k A_k^{\dagger} A_k = 1$.

If $\mathcal{B}\left(\mathcal{H}^{\mathcal{S}}\right)$ is the algebra of all operators acting on $\mathcal{H}^{\mathcal{S}}$, the *interaction algebra* $\mathcal{A}^{\text{int}} \subseteq \mathcal{B}\left(\mathcal{H}^{\mathcal{S}}\right)$ is the subalgebra generated by the A_k (assuming that $\mathcal{A}^{\text{int}}$ is closed under $\dagger$ and ϕ is unital). Up to a unitary transformation, $\mathcal{A}^{\text{int}}$ can be written as a direct sum of $d_j \times d_j$ complex matrix algebras, each of which appear with multiplicity μ_j:

$$\mathcal{A}^{\text{int}} \simeq \bigoplus_j \mathbf{1}_{\mu_j} \otimes \mathcal{B}\left(\mathbf{C}^{d_j}\right), \tag{27}$$

where $\mathbf{1}_{\mu_j}$ is the identity operator on $\mathbf{C}^{\mu_j}$.

The *commutant*, $\mathcal{A}^{\text{int}'}$ of $\mathcal{A}^{\text{int}}$ is the set of all operators in $\mathcal{B}\left(\mathcal{H}^{\mathcal{S}}\right)$ that commute with every element of $\mathcal{A}^{\text{int}}$, i.e.,

$$\mathcal{A}^{\text{int}'} \simeq \bigoplus_j \mathcal{B}\left(\mathbf{C}^{\mu_j}\right) \otimes \mathbf{1}_{d_j}. \tag{28}$$

This decomposition induces a natural decomposition of $\mathcal{H}^{\mathcal{S}}$:

$$\mathcal{H}^{\mathcal{S}} = \bigoplus_j \mathbf{C}^{\mu_j} \otimes \mathbf{C}^{d_j}. \tag{29}$$

Note now that any state ρ in $\mathcal{A}^{\text{int}'}$ is a fixed point of ϕ since it commutes with all the A_k:

$$\phi\left[\rho^{\mathcal{S}}\right] = \sum_k A_k \rho^{\mathcal{S}} A_k^{\dagger} = \sum_k A_k A_k^{\dagger} \rho^{\mathcal{S}} = \rho^{\mathcal{S}}. \tag{30}$$

It can be shown [12] that the reverse also holds, i.e.,

$$\phi\left[\rho^{\mathcal{S}}\right]=\rho \quad \Leftrightarrow \quad \rho \in \mathcal{A}^{\mathrm{int}\prime}. \tag{31}$$

Hence, the noiseless subsystem can be identified with the $\mathbf{C}^{\mu_j}$ in equation (29).

These are relevant for the physical description of the system, because the interactions with the environment will not distrurb them and hence are useful for describing the long term behavior of the system, because they are conserved. The relevance of this argument for quantum gravity was proposed in [13]. If we divide the quantum state of the gravitational field arbitrarily into subsystems, those properties which are conserved under interactions between the subsystems are going to characterize the low energy limit of the spacetime geometry. If classical spacetime physics emerges in the low energy limit, in which these regions are arbitrarily large on Planck scale, the commutant of the interaction algebra should include the symmetries that characterize classical spacetime in the ground state (presumably the Poincaré or the deSitter group, or some deformation of them). In the case of vanishing cosmological constant, we then expect that the low energy limit is characterized by representations of the Poincaré group. But these, of course, are the elementary paticles, as described in quantum field theory on Minkowski spacetime. The noiseless subsystem idea is then a way to understand how particle states, and the low energy vacuum could emerge from a background independent quantum theory of gravity.

In the following section, the system state space is that of the states of loop quantum gravity that correspond to a black hole of a given area. This is coupled to an environment of bulk spin network states and, while we do not know the quantum dynamics, we do know that a Schwarzchild black hole has a classical SO(3) symmetry. In the next section, we make the reasonable assumption that there is a microscopic analogue of SO(3) that acts on the system, a symmetry that should be contained in the commutant of the interaction algebra for black holes.

3 Application to black holes

In this section we shall look for the subspace $\mathcal{H}_A^{\text{Schwarzschild}}$ of states that correspond to a classical Schwarzschild black hole. We want to show that $\mathcal{H}_A^{\text{Schwarzschild}}$ is a proper subspace of the space $\mathcal{H}_A^{\text{All}}$ of states of horizons with area $A \pm \Delta A$, i.e.,

$$\mathcal{H}_A^{\text{Schwarzschild}} \subsetneq \mathcal{H}_A^{\text{All}}. \tag{32}$$

The geometry of a horizon is only partially fixed by its area A. A Schwarzschild horizon is a very special example of such a geometry that is distinguished by its SO(3) symmetry. Since all the other horizon geometries have quantum counterparts in $\mathcal{H}_A^{\text{All}}$ as well, we expect that the Hilbert space $\mathcal{H}_A^{\text{Schwarzschild}}$ corresponding to a Schwarzschild black hole is a proper subspace of $\mathcal{H}_A^{\text{All}}$.

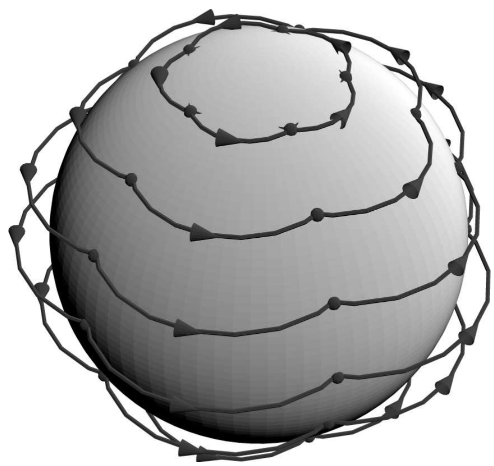

Figure 1: The geometry of a quantum horizon is concentrated at a discrete set of punctures. The classical symmetry group SO(3) will thus not act directly on the quantum states representing a Schwarzschild black hole. A discrete group G_q, on the other hand, is expected to act on these states. In the classical limit, the action of this discrete group coincides with that of SO(3). For a large number of punctures, the discrete group action shown above would approximate a rotation of the horizon.

In contrast to the smooth horizon geometry of a classical Schwarzschild black hole, the geometry of the corresponding quantum horizon is concentrated at the punctures and is thus discrete. Consequently, we cannot expect an action of the smooth Lie group SO(3) on $\mathcal{H}_A^{\text{Schwarzschild}}$. What we can expect is an action of some discrete symmetry group G_q that only on the classical level coincides with the action of SO(3) (see Figure 1).

What can reasonably be assumed about the group G_q?

Since G_q is a group of symmetries we assume it commutes with the action of the Hamiltonian H^{int} on the horizon Hilbert space $\mathcal{H}^S$, i.e.,

$$G_q \subset \mathcal{A}^{\text{int}\prime}. \tag{33}$$

We do not know the precise action of the Hamiltonian H^{int} but it is possible to restrict it sufficiently to compute the commutant G_q.

We first review the basics of the description of black hole horizons in loop quantum gravity[5]. The Hilbert space is at the kinematical level of the form (22), where the environmental Hilbert space has a basis consisting of all spin networks that end on the horizon, in any number of punctures. Given that the energy of a black hole is a function of its area, it makes sense to decompose the Hilbert space in eigenspaces of the area operator. The eigenvalues are given by sets of spins at the punctures $\{j_i\}$, where i labels the punctures. Gauss's law acting at the boundary enforces

that the labels on the punctures on the horizon match the spins of the edges of spin network in the bulk, incident on the horizon. We then have,

$$\mathcal{H}^{total} = \sum_{\{j_i\}} \mathcal{H}^{\mathcal{S}}_{\{j_i\}} \otimes \mathcal{H}^{\mathcal{E}}_{\{j_i\}}, \tag{34}$$

The environment (sometimes called "bulk") Hilbert spaces $\mathcal{H}^{\mathcal{E}}_{\{j_i\}}$ have bases consisting of diffeomorphism classes of spin networks in the bulk, with edges ending on the boundary at the punctures with the specified spin labels. We note that the diffeomorphisms on the surface are fixed. (There may also be an exterior boundary at which the diffeomorphisms are also restricted.)

The system, or surface, Hilbert spaces $\mathcal{H}^{\mathcal{S}}_{\{j_i\}}$ are direct sums of the one dimensional Hilbert spaces of $U(1)$ Chern-Simons theory on the sphere, with punctures labeled by charges m_i at points $\sigma_i \in S^2$, subject to the conditions

$$|m_i| \leq j_i, \tag{35}$$

$$\sum_i m_i = 0. \tag{36}$$

The level k of each Chern-Simons theory depends on the j_i's and is given by [5],

$$k = \frac{a_0[\{j_i\}]}{4\pi\gamma} \tag{37}$$

where $a_0[\{j_i\}]$ is the nearest number to $A[\{j_i\}]$ such that the level k is an integer. We note that the level k is very large for black holes large in Planck units, so that for some estimates the limit $k \to \infty$ can be taken. This is normally done in the computation of the entropy, for which the difference between the classical and quantum dimensions may be neglected.

The $U(1)$ connection on the boundary satisfies

$$F_{ij}(\sigma) = \frac{4\pi}{k} \sum_i m_i \delta^2(\sigma, \sigma_i). \tag{38}$$

We then can use a basis of the boundary theory, which is labeled

$$|(j_1, m_1), (j_2, m_2), \ldots, (j_N, m_N)\rangle. \tag{39}$$

We can now turn to the specification of the interaction algebra. We begin by specifying a general interaction algebra, which consists of all possible interactions between an environment and a quantum horizon. Then we will specialize to a subalgebra which excludes the case of an asymmetric environment or external geometry. The general interaction algebra, $\mathcal{A}^{\text{gint}}$, must be generated by operators that

1. Act simultaneously on the environment and system Hilbert space.

2. Act locally.

A sufficient set of generators for $\mathcal{A}^{\text{gint}}$ consists of:

- Addition or removal of a puncture.

- Braidings of the punctures.

The Gauss's law constraint that ties the labels on punctures to the labels on edges of spin networks that meet them implies that each of these involve changes to both the surface and environment state. We note that the braidings generate a group, called the braid group and together with adding and removing punctures, these generate the tangle algebra discussed by Baez in [15].

Let us consider, for example, a generator in $\mathcal{A}^{\text{gint}}$ that corresponds to braiding two punctures, which we will label $i = 1, 2$. It has the effect of rotating, with a positive orientation, the two punctures around each other, returning them to their original positions. This braids the two edges of the bulk spin network, which changes the diffeomorphism class of the bulk state.

The action of H^{int} on $\mathcal{H}^{\mathcal{S}}$ due to a braiding of the punctures can be shown to modify each basis state (39) by a phase. As the punctures are unchanged, it is represented by an operator $\hat{\mathcal{B}}_{12}$ in $\mathcal{H}^{\mathcal{S}}_{\{j_i\}}$. A simple computation in the quantum Chern-Simons theory [14] shows that this is realized by

$$\hat{\mathcal{B}}_{12} \circ |(j_1, m_1), (j_2, m_2), \ldots, (j_N, m_N)\rangle = e^{\frac{2\pi \imath (m_1 + m_2}{k}} |(j_1, m_1), (j_2, m_2), \ldots, (j_N, m_N)\rangle. \tag{40}$$

The general interaction algebra does not take into account any symmetries, hence it describes the general case, in which the black hole horizon may have non-vanishing multiple moments[4]. If we want to specialize to the case of a black hole with symmetries, we must impose conditions on the interaction algebra, for a symmetric subsystem cannot exist stably in an asymmetric environment. In principle we could code various kinds of symmetries in the choice of interaction algebra. But it suffices to consider the simplest choice, which is the subalgebra $\mathcal{A}^{\text{int}} \subset \mathcal{A}^{\text{gint}}$ which consists of adding and removing punctures plus a symmetric sum of braiding operations,

$$\hat{\mathcal{B}}^T = \sum_{i<j} \hat{\mathcal{B}}_{ij} \tag{41}$$

where the sum is over all pairs of punctures.

To restrict the group G_q, we now assume that it shares two properties with its classical counterpart SO(3). The first property is:

[4]In the isolated horizon approach[5] angular momentum and multiple moments are coded as classical parameters. Here we seek instead to define different subspaces of the horizon Hilbert space associated with black holes with different macroscopic properties.

P1 The elements of G_q do not change the area of the horizon.

We take this to mean that the elements of G_q commute with the area operator[5]. This is trivially true for the action of SO(3) on the classical spacetime and we assume that it is also true for the action of G_q. Because of the property **P1**, the elements of G_q will map the spaces $\mathcal{H}^{\mathcal{S}}_{\{j_i\}}$ into themselves. A necessary condition for equation (33) to hold, i.e., for G_q to commute with the action of H^{int}, is that G_q commutes with the action of H^{int} on each of the spaces $\mathcal{H}^{\mathcal{S}}_{\{j_i\}}$.

On each $\mathcal{H}^{\mathcal{S}}_{\{j_i\}}$ the braiding generators in H^{int} act as shown in equation (40). The commutant of $\mathcal{A}^{\text{int}}$ is then easy to find. It is generated by all those operators that just permute the m_i values for given spins j_i:

$$|\ldots,(j,m_1),\ldots(j,m_K),\ldots\rangle \longrightarrow |\ldots,(j,m_{\pi(1)}),\ldots(j,m_{\pi(K)}),\ldots\rangle, \tag{42}$$

for some $\pi \in P_K$, the permutation group of K objects. Assuming G_q to be unitary we then have

$$G_q \subset \prod_j P_{K_j}, \tag{43}$$

where the product is over all the different j's in $\{j_i\}$.

We now assume that G_q also shares the following property with SO(3):

P2 G_q acts transitively on the punctures, i.e., for every two punctures there is an element in G_q that connects the two.

Because of equation (43), we know that G_q is a subgroup of

$$P = \prod_j P_{K_j}. \tag{44}$$

For G_q to act transitively, it is necessary that the bigger group P acts transitively on the punctures. Since P is the product of permutation groups, this is only possible if P coincides with just one permutation group. It follows that there is one j such that

$$K_j = N. \tag{45}$$

All the punctures have the same spin j and the Hilbert space $\mathcal{H}^{\text{Schwarzschild}}_A$ consists of those $\mathcal{V}_{\{j\}}$ for which all the j's coincide. It is then straightforward to show that the dimension of the $\mathcal{V}_{\{j\}}$ varies sharply with the spin j and is dominated by the lowest spin j_{min}.

[5] In principle we could consider a weaker requirement which is that the elements of G_q do not change the expectation value of the area. This would be an interesting extension of the usual problem to study.

4 Conclusions

In this paper, we used a microscopic analogue of the classical SO(3) symmetry of a Schwartzchild black hole to restrict the loop quantum gravity state space $\mathcal{H}_A$ of black holes of area A to a smaller subspace, left invariant by the symmetry. The method we used is the noiseless subsystems of quantum information theory, in which the symmetry of the dynamics implies a non-trivial commutant of the interaction algebra of the system.

The construction of the state space $\mathcal{H}_A$ in loop quantum gravity is a hybrid construction that requires the imposition of black hole horizon conditions at the classical level. One eventually wants to be able to identify black hole states directly in the quantum theory and derive the classical geometry in the appropriate classical limit. While the present work has the same hybrid character (we start from $\mathcal{H}_A$), it indicates that properties of the quantum states can be inferred from the dynamics algebraically, without need for a classical geometry.

This approach also clarifies the important but subtle question of how symmetries can arise from a diffeomorphism invariant state. All the states discussed here are invariant under spatial diffeomorphisms. The point is that the commutant, G_q takes physical, diffeomorphism invariant states, to other distinct physical, diffeomorphism invariant, states. The transitive permutations that we discuss translate the black hole horizon with respect to the spin networks that define the external geometry. This is physically meaningful because these transformations are not subgroups of the diffeomorphisms, instead they act on the space of diffeomorphism invariant states. Thus, we have a realization of the proposal in [13] that in a diffeomorphism invariant theory a symmetry can only arise as a motion of one subsystem with respect to another, and its generators must live in the commutant of the interaction algebra, defined by the splitting of the universe into subsystems.

This proposal has ramifications that might lead to a better understanding of how the classical properties of black holes arise from the quantum geometry of the horizon. We also see in a simple example, how properties of classical spacetime geometries can emerge from exact quantum geometries, by making use of the insights gained from the study of similar questions in quantum information theory.

We close by mentioning several queries that may be made regarding the construction used here.

- *What about the connection to quasi-normal modes? Can we understand it as other than a coincidence?* It may be possible to understand how the quasi-normal modes arise from the states which transform non-trivially under those generators of the commutant that become rotations in the $A \to \infty$ limit.

- *What about rotating black holes?* It is of interest to use these methods to characterize the states of a rotating black hole, or a horizon with multiple moments, in terms of the same language. We expect that this will involve other choices

of the interaction algebra than that made here. We may note that attempts to extend the connection between quasi-normal modes and entropy to rotating black holes have not succeeded; this is something that needs explanation.

Note also that while we employ here the same boundary conditions as the isolated horizon formalism, the intension is different. In particular, in the isolated horizon picture, rotation and other multiple moments are fixed in the classical description. The isolated horizon boundary conditions don't change, but the functions on the phase space that correspond to different observables will depend on the multiple moments. In our picture, we seek to recover the generators of rotation, and hence the conjugate conserved quantities, as operators in a single Hilbert space. Hence here, unlike the pure isolated horizon case, we expect that a single Hilbert space contains the states corresponding to black holes with all values of angular momentum.

- *What about the states in $\mathcal{H}_A^{All}$ that are not in $\mathcal{H}_A^{Schwarzchild}$?* These are states whose puctures give the right area, but are not in the noiseless subsystem we identify. But they arose from a quantization of the isolated horizon boundary conditions.

  We cannot give a definitive answer without more dynamical input, particularly a Hamiltonian on the horizon states. But we can argue that they must correspond to either non-static, classical metrics or they correspond to no classical geometry.

  Given that the boundary conditions do not distinguish rotating from non-rotating black holes, some of theses states must be associated with rotating black holes. Apart from these, it must be that most of these states are associated with semiclassical states of non-stationary horizons, corresponding to distorted black holes. These will be excited, transient states of the black hole, which classically decay to stationary states, emitting gravitational radiation. How do we know that such states are included in $\mathcal{H}_A^{All}$? All that distinguishes states in $\mathcal{H}_A^{All}$ is that there is a horizon of a particular area. This will include all such configurations, whether belonging to stationary states, semiclassical states corresponding to non-stationary horizons as well as quantum states that have no semiclassical approximation. In particular, these must include states corresponding to quasi normal modes.

  Beyond those categories, our argument implies that the remaining states couple strongly to the random noise coming from the environment of the black hole. They are then states that cannot play a role in the semiclassical limit.

  The only definitive way to distinguish the different kinds of states is dynamically. If we had an effective Hamiltonian for the horizon and near horizon geometry, then the non-stationary states would have to have higher energy

than the stationary states, corresponding to their potential for radiating energy to infinity in gravitational radiation. Hence, we can deduce from the fact that in the classical theory non-stationary states radiate, that those states would be suppressed in a Boltzman weighted, equilibrium ensemble. In the ensemble $\mathcal{H}_A^{All}$ they appear unsuppressed, but that is only because no dynamics are invoked, so states of different energies are counted as having the same weight.

Given that there are many more non-stationary classical configurations than stationary ones, it is reasonable that, for a given horizon area, those states that correspond to static black holes must appear to be a minority, as is the case if we compare $\mathcal{H}_A^{All}$ with its subspace $\mathcal{H}_A^{Schwarzchild}$. But, it is incorrect to argue just on the basis of the fact that $\mathcal{H}_A^{All}$ is larger than $\mathcal{H}_A^{Schwarzchild}$, that a typical state corresponding to an equilibrium, static black hole will be in the latter rather than the former, because dynamics has yet to be invoked.

The point of the construction of this paper is then that, even in the absence of an explicit Hamiltonian, we can use symmetry properties of the Hamiltonian to apply an argument from quantum information to deduce which subspace of states will emerge in the low energy limit as corresponding to stationary configurations. The fact that, in the absence of dynamics, these are a minority of the ensemble of states with a given area must be expected and is not an objection against the construction presented here.

- Finally, we note that the same method could be applied within quantum cosmology, to study how states that correspond to symmetric universes emerge out of a more general Hilbert space, corresponding to the full theory[16].

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